

## Badobadop's Mathematics Challenge

If  $x$  and  $y$  are positive integers find a solution to the equation:

$$(3 - 2\sqrt{x})^2 = y - 6\sqrt{8}$$

### Solution

Start by multiplying out the bracket on the left hand side to give:

$$9 - 12\sqrt{x} + 4x = y - 6\sqrt{8}$$

Now collect all the integer terms on the left hand side:

$$9 + 4x - y = 12\sqrt{x} - 6\sqrt{8}$$

Note that  $6\sqrt{8} = 12\sqrt{2}$  so:

$$9 + 4x - y = 12\sqrt{x} - 12\sqrt{2}$$

If the left hand side of the equation is an integer, then the right hand side is an integer too. This means that  $12\sqrt{x} - 12\sqrt{2}$  must be an integer.

This condition is satisfied if  $x = 2$ , when:

$$12\sqrt{x} - 12\sqrt{2} = 0$$

and the right hand side of the equation is an integer.

With  $x = 2$ :

$$9 + 4x - y = 17 - y = 0$$

So  $y = 17$