Badobadop

Badobadop’s Mathematics Challenge

Last week I showed that if x and y are positive integers, and

$$\left(3-2\sqrt{x}\right)^2 = y - 6\sqrt{8}$$

Then

$$12\sqrt{x} - 12\sqrt{2} = n$$

Where n is an integer. In addition, when x = 2, n = 0, and y =17.

In order to find alternative solutions with n≠0 we rearrange

$$12\sqrt{x} - 12\sqrt{2} = n$$

To give

$$\sqrt{x} = \frac{n}{12} + \sqrt{2}$$

Squaring both sides gives

$$x = \frac{n^2}{144} + \frac{n\sqrt{2}}{6} + 2$$

So

$$x - \frac{n^2}{144} - 2 = \frac{n\sqrt{2}}{6}$$

If n is ≠ 0 we can divide both sides of the equation by n and multiply by 6 to give

$$\frac{6x}{n} - \frac{n}{24} - \frac{12}{n} = \sqrt{2}$$

Taking a common denominator of 24n on the left hand side gives

$$\frac{144x - n^2 - 288}{24n} = \sqrt{2}$$

Since x and n are integers, the left hand side of the equation is a rational number. On the right hand side, the square root of 2 is an irrational number. This is clearly not possible, so there are no solutions for which n≠0 and the solution x = 2, y = 17 must be unique.